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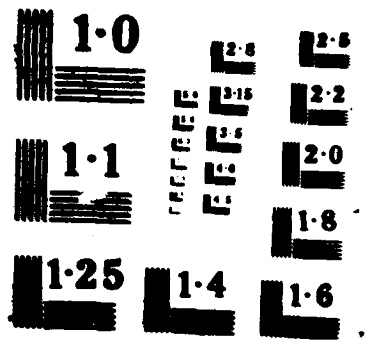
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FORCE EQUIVALENCE INDICES

US ARMY
TANK-
AUTOMOTIVE COMMAND

Mr. David J. Grant
Systems Analysis Division
Systems and Cost Analysis Directorate

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The deriving of unique scores for entities in an incomplete tournament is adapted to killer-victim matrices. These scores provide the best available comparison of weapon systems engaged in heterogeneous combat.

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Force Equivalence Indices

Mr. David J. Grant

U.S. Army Tank-Automotive Command

I. Introduction

This study was motivated by the need to revamp the computation of force ratios in the Force Comparison Model (FORCECOM). FORCECOM applied a series of multipliers to scores based on laboratory performance of the systems involved. The formulas used to derive these multipliers came from the historical analysis of Du Puy [4]. The goal in deriving Force Equivalance indices was to develop an algorithm which meets the following criteria:

1. It produces reasonable results for any killer-victim matrix to which it is applied.

2. Its mathematical formulation is the simplest possible,

3. The interpretation of the above formulation does not run counter-intuitive to equations used in the physical sciencs.

Research showed that analysts took two approaches to assigning scores to weapons. The first was to use the eigenvector corresponding to the largest eigenvalue of the adjusted killer-victim matrix. (See [2, ch. 30].) This approach, however, is limited to non-zero matrices and a certain class of non-negative matrices.

Other analysts, such as Johnsrud [5], developed non-linear methods. The ways in which they attained the desired non-linearity for their equations are not intuitively obvious. Under close examination, their algorithms violate accepted physical formulas.

The Potential-Antipotential Algorithm, developed at the U.S. Army Concepts Analysis Agency, is based on the "Job = Rate * Time' format. The first assumption made in this model is that each killer weapon type is allotted a percentage of each opposing weapon type, which is equal to the percentage of opposing weapon losses attributed to that killer. For example, if tanks accounted for 10 per cent of the opposing infantry's losses, then tanks were assigned to fight 10 per cent of the opposing infantry. This is the way the algorithm assigns opposing weapon systems

to each other. The second assumption is that the time required to conclude combat between two weapon types, with values V(i) and V(j), is 1/SQRT(V(i)*V(j)).

Solving the above equation for Time, in terms of Job and Rate, one obtains "Time = Job/Rate". Substituting the information from the first assumption, one can obtain Time = SQRT ((P(i,j)*P(j,i))/(R(i,j)*R(j,i))), where P(k,v) is the percentage of victim systems assigned to that killer type, and R(k,v) is the rate at which the victims are destroyed by the killer. There is no connection between the two expressions for time except the attempt to introduce nonlinearity.

The killer-victim matrix can serve as the set of coefficients for a system of linear difference equations. Noting this, one can see the applicability of the methods used with pulse processes, a branch of graph theory. Using ideas from this area of mathematics, the purpose of this paper is to suggest a more general linear algorithm, which satisfies the following criteria suggested by Johnsrud:

1. Arbitrary indexing of the weapons involved should not effect the results.

2. Having zero losses should not prevent solving the problem with acceptable results.

3. Killers of killers must have some weight.

4. Small changes in the scoreboard should produce small changes in the resulting values.

5. Adding either numbers or capabilities to a side should increase its computed strength relative to the opposing side.

II. Graph Theory Concepts

A directed graph is a collection of points, or nodes, together with the arcs which connect the nodes. The graph is called directed because traffic along an arc is one-way. A path is a set of continuous arcs leading from one node to another. Two nodes are connected if there is a path from one to the other.

A set of nodes is said to be strongly connected if each member node has a path to every other node in the set.

A strongly connected set is called a strong component if it contains all the nodes in a digraph which are strongly connected to the nodes in the set. Strong components partition the digraph which are strongly connected to the nodes in the set. Strong components partition the digraph's nodes into mutually exclusive sets. If these stong components are treated as nodes, a new digraph, called a condensation, can be derived. The condensation's arcs are found by connecting strong components with an arc if an only if a member node of one strong component is connected to a member node of the other strong component.

The killer-victim matrix for a battle has a directed graph associated with it. The nodes are the weapon system classes, and the weighted arcs are the numbers of the systems at the initial nodes "killed" per system at the terminal node. The strong components partition the collection of weapon system classes into groups of weapon systems which interact with each other and which should be treated as separate problems. Once the strong components have been identified, the killer-victim matrix should be converted into one of the coefficient matrices discussed under "Coefficients".

The following technique assumes that there is a fixed amount of "value" in any battle.

1. Partition the systems into strong components.

2. Construct the condensation of the systems.

3. Beginning with an estimate of value for each strong component, multiply the estimate vector, xV, by the adjusted condensation matrix, V[*], until the maximum change in the vector's values is sufficiently close to zero. Use this last estimate of values.

4. Using the values obtained for the condensation, assign a percentage of the total value to each strong component.

5. Find the associated vectors for each strong component and convert to percentages within strong components.

6. Compute the percentage of total value per weapon system by multiplying the weapon's percentage of strong component value by strong component's percentage of total value.

If the battle has an irreducible matrix representation, it is itself the only strong component. The case then devolves to solving by the eignvalue technique, using the coefficient matrix.

Another useful concept is that of the outscore, or the sum of entries in a row of a matrix. It is usually used to rank the participants of a tournament, the matrix involved having only 0's and 1's. If the percentage of matches won is substituted for the 1=win and 0=loss of the tournament, one obtains the first interation of the multiplication described above.

Since weapons in each force generally do not inflict losses on each other, the tournament resulting from the killer-victim matrix is incomplete. Jech [6] proves that there is a unique completion of this tournament. Each V(i,j) is replaced by P(i,j)*M(i,j), where the following hold:

1. M(i,j) = M(j,i) is the number of "matches" between i-type weapons and j-type weapons.

2. P(i,j) + P(j,i) = 1, if i $\neq$ j.

3. P(k,i)*P(i,j)*P(j,k) = P(k,j)*P(j,i)*P(i,k).

If P(i,j) is set equal to u(i)/(u(i)+u(j)), one can derive a system of N nonlinear equations in N variables (F). Its solution can be approximated by the following equation:

u(i:n+1) = u(i:n) *EXP(s(i) - F(i:u(*:n))), where s(i) is

the incomplete outscore of the i-th weapon type, and F(i:u(*:n)) is the i-th component of the nonlinear transformation on the n-th iteration of the vector u.

III. Coefficients

There are two main divisions in the types of coefficients used in this paper: lethality and survivability. Lethality scores are basicaly a percentage killed during the observed "combat", or the percentage expected to be killed at some calculated time. Survivability scores are the percentage surviving and are calculated by taking the transpose of the complement of the lethality score. (A list of descriptions can be found in [11].)

The "completed" outscores have a built-in mixture of lethality and survivability. If the weapon killed, it survived. If it lost, it died. This is not true in the case of the vector convergence method. For that reason, all of these coefficients will be evaluated with the vector convergence method, while the outscore completion will use only the percent killed.

IV. Methodology

The coefficients in the previous chapter were tested with the data found in Table 1. The following is a list of the tests used to check compliance with Johnsrud's criteria.

1. Arbitrary indexing is permitted.

    a. In the case of vector convergence, a dummy vector is used during matrix multiplication, and this allows for interchange of rows and columns.

    b. The formation of condensations and the computation of outscores is unaffected by the interchange of rows and columns.

2. Reducible matrices give reasonable results.

    a. In the case of vector convergence, the sequence of vectors does not diverge.

    b. Where condensation is used, strong components which are intuitively more effective will have larger percentages of battle value.

    c. Intuitively more effective systems will have larger outscores than other systems.

3. Killers of killers should have non-zero values. This is met by all three algorithms.

4. Small scoreboard changes should produce small changes in results.

    a. For each side, infantry is allowed to kill one artillery piece.

    b. Changes in the force ratio will be less than five percent.

5. Adding something to a side increases its worth relative to the other side.

a. The initial number of Blue armor was increased ten percent.

b. The force ratio (FR = R/B) will decrease.

V. Results

The results of the tests for the coefficients, using vector conversance, are in Table 2. Only three coefficients passed all criteria and those were all survivability scores. The "completed" [illegible] ores also passed all of the criteria. (See Table 2.)

As a side [illegible], two versions of the Potential-Antipotential Algorithm were checked as well. The original version failed to lower the force ratio when Blue's armor was increased. The second version, which converted the matrix entries to percentage killed per system instead of per category, passed all criteria. (See Table 2.)

VI. Analysis

Of the methods tested, the completed outscores have the strongest appeal. Its mathematical founding is the surest, and it satisfies the largest number of the criteria listed in this paper.

It is easy to prove that Criterion #4 will hold for the completed outscores. Suppose that the initial number of weapon system x has been increased by a factor of $A > 1$. The incomplete outscores of systems friendly to x will remain the same since none of them killed x. Weapon x's incomplete outscore will decrease to $S(x)/A$. The $P(x,j)$'s will remain the same, however, since $1/A$ can be factored out of the new complete equation for x to get the old one. Therefore, the $P(j,x)$'s must also remain the same.

Suppose an opposing weapon system, say z, killed on x-type weapon. Then, z's incomplete outscore will decrease, forcing a decrease in at least one $P(z,k)$ and a corresponding increase in $P(k,z)$. The value of the side including x and k will increase, and the opposing side's value will decrease, producing the desired change in the force ratio.

If no opposing system killed weapon x, the opponent's value will remain the same, as no $P(i,j)$'s would change, but the value of x's side would still increase. Thus, the criterion is satisfied.

To prove that Criterion #5 will hold, consider the following equation.

$$FR(delta) = FR(old) - FR(new)$$

$$= \frac{(Vsum(old)*Ksum(new)) - (Vsum(new)*Ksum(old)}{Ksum(old) * Ksum\ (new)}$$

Suppose that an entry in the killer-victim matrix is changed from L to L+D. The new number of matches between the killer and victim becomes

$$M'(k,v) = M'(v,k) = M(k,v) + E, \text{ where } E = D/(S(k)*S(v)).$$

Only two changes are required in the P(i,j)'s.

$$P'(k,v) = \frac{(P(k,v)*M(k,v)) + E}{M(k,v) + E}, \text{ and}$$

$$P'(v,k) = \frac{(1-P(k,v))*M(k,v)}{M(k,v) + E},$$

The new completed outscores for killer and victim are:

$$CS'\ (k) = CS(k) + (E*(1 - P(k,v))/(M(k,v) + E), \text{ and}$$

$$CS'\ (v) + CS(v) - (E*(1 - P(K,v)).$$

The values for Ksum(new) and Vsum(new) become:

$$Ksum(new)=Ksum(old)+((I(k)*E*(1-P(k,v))/(M(k,v)+E), \text{ and}$$

$$Vsum(new) = Vsum(old) - (I(v)*E*(1 - P(k,v)).$$

Taking the limit of both sides as E goes to zero, the equation for FR(delta) becomes:

$$FR(delta) = \frac{Vsum(old)*Ksum(old) - Vsum(old)*Ksum(old)}{Ksum(old)*Ksum(old)} = 0.$$

Thus, Criterion #5 is established for the completed outscores.

There is a class of matrices which escapes all of the methods examined. If the forces can be partitioned into two or more mutually exclusive battles, the figures given

by these algorithms will have to be justified on a non-mathematical basis, such as battle conditions or the unusual tactics employed. In general, the opposing forces in these mutually exclusive "sub-battles" should be evaluated separately.

VII. Conclusions

First, the present methods of computing weapon system values (the eigenvalue methods and the Potential-Antipotential Algorithm) have serious drawbacks, both from a mathematical view and a practical view, and should be replaced.

Second, the method of completed outscores provides an intuitively pleasing approach which is mathematically sound and which is proven to satisfy the acceptance criteria.

Table 1. Augmented Killer-Victim Scoreboard

		VICTIMS									
		BLUE					RED				
		Inf	Arm	ADA	AH	FA	Inf	Arm	ADA	AH	FA
	Inf	110	45	10	10	0	0	0	0	0	0
	R Arm	100	85	5	0	0	0	0	0	0	0
K	E ADA	0	0	0	10	0	0	0	0	0	0
I	D AH	30	15	15	5	0	0	0	0	0	0
L	FA	70	15	20	0	10	0	0	0	0	0
L											
E	Inf	0	0	0	0	0	100	30	10	0	0
R	B Arm	0	0	0	0	0	100	80	20	0	0
S	L ADA	0	0	0	0	0	0	0	0	5	0
	U AH	0	0	0	0	0	50	20	15	3	0
	E FA	0	0	0	0	0	80	10	20	0	5
Initial Systems		1200	300	200	50	50	1000	500	100	10	60

Table 2. FEIs and Force Ratios

	Weapon Scores										
	BLUE					RED					
Case No.	INF	ARM	ADA	AH	FA	INF	ARM	ADA	AH	FA	Force Ratio
Completion											
1	1.	0.98	1.19	1.69	1.98	1.03	1.21	0.91	2.11	2.17	.98
2	1.	1.01	1.18	1.69	1.99	1.04	1.22	0.92	2.11	2.18	.97
3	1.	0.98	1.18	1.68	1.96	1.03	1.21	0.9	2.1	2.15	.981
Passed all criteria.											
Potential - Antipotential											
1	1.	6.98	11.3	38.1	37.1	3.	5.77	17.7	163.	53.1	1.339
2	1.	6.33	11.4	38.3	37.3	3.01	5.6	18.1	166.	54.1	1.340
3	1.	6.7	10.9	36.6	34.6	2.91	5.53	17.	156.	49.5	1.330
Failed criterion #5 since the FR change in #2 favors Red.											
Altered Potential - Antipotential											
1	1.	7.57	35.4	94.4	49.8	1.69	2.19	13.5	92.9	32.2	.934
2	1.	6.87	35.4	94.5	49.9	1.65	2.	13.4	91.5	31.9	.384
3	1.	7.03	32.8	87.6	46.1	1.6	2.04	12.5	86.4	30.	.395
Passed all criteria.											
Vector Convergence											
1	1.	7.66	164.	426.	143.	16.9	2.06	152.	994.	285.	.93
Failed criterion #2 since artillery was rated less than other systems. Other criteria not tested.											

END

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